Contents

Preface vii
Preface to the First Edition xiv

Part I Introduction

1 Background and Overview 3
1.1 Background 3
1.2 Overview 4

2 Casting Models in Canonical Form 9
2.1 Notation 9
2.1.1 Log-Linear Model Representations 11
2.1.2 Nonlinear Model Representations 11
2.2 Linearization 12
2.2.1 Taylor Series Approximation 12
2.2.2 Log-Linear Approximations 14
2.2.3 Example Equations 15

3 DSGE Models: Three Examples 18
3.1 Model I: A Real Business Cycle Model 20
3.1.1 Environment 20
3.1.2 The Nonlinear System 23
3.1.3 Log-Linearization 26
3.2 Model II: Monopolistic Competition and Monetary Policy 28
3.2.1 Environment 28
3.2.2 The Nonlinear System 33
3.2.3 Log-Linearization 34
3.3 Model III: Asset Pricing 38
3.3.1 Single-Asset Environment 38
3.3.2 Multi-Asset Environment 39
3.3.3 Alternative Preference Specifications 40

Part II Model Solution Techniques

4 Linear Solution Techniques 51
4.1 Homogeneous Systems 52
4.2 Example Models 54
4.2.1 The Optimal Consumption Model 54
4.2.2 Asset Pricing with Linear Utility 55
4.2.3 Ramsey's Optimal Growth Model 56
4.3 Blanchard and Kahn’s Method 57
4.4 Sims’ Method 61
4.5 Klein’s Method 64
4.6 An Undetermined Coefficients Approach 66

5 Nonlinear Solution Techniques 69
5.1 Projection Methods 71
  5.1.1 Overview 71
  5.1.2 Finite Element Methods 72
  5.1.3 Orthogonal Polynomials 73
  5.1.4 Implementation 74
  5.1.5 Extension to the \( I \)-dimensional Case 78
  5.1.6 Application to the Optimal Growth Model 79
5.2 Iteration Techniques: Value-Function and Policy-Function Iterations 87
  5.2.1 Dynamic Programming 87
  5.2.2 Value-Function Iterations 89
  5.2.3 Policy-Function Iterations 94
5.3 Perturbation Techniques 95
  5.3.1 Notation 95
  5.3.2 Overview 97
  5.3.3 Application to DSGE Models 99
  5.3.4 Application to an Asset-Pricing Model 105

Part III Data Preparation and Representation

6 Removing Trends and Isolating Cycles 113
  6.1 Removing Trends 115
  6.2 Isolating Cycles 120
    6.2.1 Mathematical Background 120
    6.2.2 Cramér Representations 124
    6.2.3 Spectra 125
    6.2.4 Using Filters to Isolate Cycles 126
    6.2.5 The Hodrick-Prescott Filter 128
    6.2.6 Seasonal Adjustment 130
    6.2.7 Band Pass Filters 131
  6.3 Spuriousness 134

7 Summarizing Time Series Behavior When All Variables Are Observable 138
  7.1 Two Useful Reduced-Form Models 139
    7.1.1 The ARMA Model 139
    7.1.2 Allowing for Heteroskedastic Innovations 145
    7.1.3 The VAR Model 147
Contents

7.2 Summary Statistics 149
  7.2.1 Determining Lag Lengths 157
  7.2.2 Characterizing the Precision of Measurements 159
7.3 Obtaining Theoretical Predictions of Summary Statistics 162

8 State-Space Representations 166
  8.1 Introduction 166
    8.1.1 ARMA Models 167
  8.2 DSGE Models as State-Space Representations 169
  8.3 Overview of Likelihood Evaluation and Filtering 171
  8.4 The Kalman Filter 173
    8.4.1 Background 173
    8.4.2 The Sequential Algorithm 175
    8.4.3 Smoothing 178
    8.4.4 Serially Correlated Measurement Errors 181
  8.5 Examples of Reduced-Form State-Space Representations 182
    8.5.1 Time-Varying Parameters 182
    8.5.2 Stochastic Volatility 185
    8.5.3 Regime Switching 186
    8.5.4 Dynamic Factor Models 187

Part IV Monte Carlo Methods 193

9 Monte Carlo Integration: The Basics 193
  9.1 Motivation and Overview 196
  9.2 Direct Monte Carlo Integration 198
    9.2.1 Model Simulation 198
    9.2.2 Posterior Inference via Direct Monte Carlo Integration 201
  9.3 Importance Sampling 202
    9.3.1 Achieving Efficiency: A First Pass 206
  9.4 Efficient Importance Sampling 211
  9.5 Markov Chain Monte Carlo Integration 215
    9.5.1 The Gibbs Sampler 216
    9.5.2 Metropolis-Hastings Algorithms 218

10 Likelihood Evaluation and Filtering in State-Space Representations Using Sequential Monte Carlo Methods 221
  10.1 Background 221
  10.2 Unadapted Filters 224
  10.3 Conditionally Optimal Filters 228
  10.4 Unconditional Optimality: The EIS Filter 233
    10.4.1 Degenerate Transitions 235
    10.4.2 Initializing the Importance Sampler 236
  10.4.3 Example 239
10.5 Application to DSGE Models
  10.5.1 Initializing the Importance Sampler
  10.5.2 Initializing the Filtering Density
  10.5.3 Application to the RBC Model

Part V Empirical Methods

11 Calibration
  11.1 Historical Origins and Philosophy
  11.2 Implementation
  11.3 The Welfare Cost of Business Cycles
  11.4 Productivity Shocks and Business Cycle Fluctuations
  11.5 The Equity Premium Puzzle
  11.6 Critiques and Extensions
    11.6.1 Critiques
    11.6.2 Extensions

12 Matching Moments
  12.1 Overview
  12.2 Implementation
    12.2.1 The Generalized Method of Moments
    12.2.2 The Simulated Method of Moments
    12.2.3 Indirect Inference
  12.3 Implementation in DSGE Models
    12.3.1 Analyzing Euler Equations
    12.3.2 Analytical Calculations Based on Linearized Models
    12.3.3 Simulations Involving Linearized Models
    12.3.4 Simulations Involving Nonlinear Approximations
  12.4 Empirical Application: Matching RBC Moments

13 Maximum Likelihood
  13.1 Overview
  13.2 Introduction and Historical Background
  13.3 A Primer on Optimization Algorithms
    13.3.1 Simplex Methods
    13.3.2 Derivative-Based Methods
  13.4 Ill-Behaved Likelihood Surfaces: Problems and Solutions
    13.4.1 Problems
    13.4.2 Solutions
  13.5 Model Diagnostics and Parameter Stability
  13.6 Empirical Application: Identifying Sources of Business Cycle Fluctuations

14 Bayesian Methods
  14.1 Overview of Objectives
  14.2 Preliminaries