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# Data Envelopment Analysis

Second Edition

# DATA ENVELOPMENT ANALYSIS

A Comprehensive Text with Models,  
Applications, References  
and DEA-Solver Software

Second Edition

WILLIAM W. COOPER

University of Texas at Austin, U.S.A.

LAWRENCE M. SEIFORD

University of Michigan, U.S.A.

KAORU TONE

National Graduate Institute for Policy Studies, Japan

William W. Cooper  
University of Texas, USA

Lawrence M. Seiford  
University of Michigan, USA

Kaoru Tone  
National Graduate Institute for Policy Studies, Japan

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**Ruth, Bev and Michiko**

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